



Trajectory Optimization for Thermally-Actuated Soft Planar Robot Limbs

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A. Wertz, A. P. Sabelhaus, and C. Majidi, "Trajectory optimization for thermally-actuated soft planar robot limbs," in IEEE 5th International Conference on Soft Robotics (RoboSoft). 2022.

Project at a glance

- Sensorizing a compliant, thermally actuated robot limb
- Develop simplified system model
- Calibrate model parameters
- Trajectory optimization for open-loop control
- Validate in hardware
- Future directions

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Overview

Background, Hardware, Experiment







SML

Soft systems are hard to model

- Kinematics hard "Infinite" DoF
- Dynamics hard Continuum deformation and rate-dependent mechanics

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Thermal actuation of shape memory alloy (SMA) is hard to model

- Difficult/impossible to measure metallurgical state
- Difficult to deal with hysteresis

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Open-loop control not well-explored

- Since modeling is difficult, mostly just use some form of sensing and control
- Not as good for untethered applications or planning

Hardware configuration



A. Cast silicone limb mounted to test fixture

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B. Exploded view of limb (1), SMA actuators (2), thermocouples (3), bend sensor (4), and mounting bracket (5)

C. Top down view of actuation and bend angle

Data flow and experimental design



D. Data flow diagram

E. Experimental design



Dynamics Modeling

Continuum limb, Thermal actuator





- Used typical rigid link manipulator model
 - Alternatives exist, namely DER



 $\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + k\boldsymbol{\theta} + \sigma\dot{\boldsymbol{\theta}} = \mathbf{f}(\mathbf{T}).$



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- Added:
 - Torsional spring at joints



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 - Viscous damping
 - Actuator torques (next)

 E_2 φ E_1 φ θ_3 $r_3(\theta) \cdot E_1$

 $\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + k\boldsymbol{\theta} + \sigma\dot{\boldsymbol{\theta}} = \mathbf{f}(\mathbf{T})$



• Crystal lattice deformations translate to macroscopic strain



Wikipedia contributors. "Shape-memory alloy." *Wikipedia, The Free Encyclopedia.* Wikipedia, The Free Encyclopedia, 7 Apr. 2022. Web. 25 Apr. 2022.



- Crystal lattice deformations translate to macroscopic strain
 - Shape memory effect



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Single actuation path



Lagoudas, Dimitris C., ed. *Shape memory alloys: modeling and engineering applications.* Springer Science & Business Media, 2008.



Austenite

typical NiTi SMA.

- Crystal lattice deformations translate to macroscopic strain
 - Shape memory effect 0
 - Pseudoelasticity 0



- Crystal lattice deformations translate to macroscopic strain
 - Shape memory effect
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Challenges

 Hysteretic effects on stress- and temperature- induced deformations



Μ,

M

Α,

Fig. 1.7. A pseudoelastic loading path.

Fig. 1.10. Stress-strain-temperature data exhibiting the shape memory effect for a typical NiTi SMA.

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Temperature, T



Twinned Martensite

Austenite

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Challenges

- Hysteretic effects on stress- and temperature-induced deformations
- Transformation temperatures change based on stress
- Martensite fractions difficult to measure



typical NiTi SMA.





Ignore hysteresis



Ignore hysteresis

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Convective heat transfer model: (ignoring effect of martensite fraction)



Ignore hysteresis

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Ignore hysteresis

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Convective heat transfer model: (ignoring effect of martensite fraction)



Linear force mapping:

$$\mathbf{f}(\mathbf{T}) = \beta_r (T_r - T_0) \mathbb{1}_{\mathbf{n}} - \beta_l (T_l - T_0) \mathbb{1}_{\mathbf{n}}.$$

Ignore hysteresis

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Convective heat transfer model: (ignoring effect of martensite fraction)



Linear force mapping:

 $\mathbf{f}(\mathbf{T}) = \beta_r (T_r - T_0) \mathbb{1}_{\mathbf{n}} - \beta_l (T_l - T_0) \mathbb{1}_{\mathbf{n}}.$

- Reasonable for some operating conditions, not all
- Ignores martensite fraction
- Assumes linear relationship between martensite fraction and temperature



Control inputs













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Kinematics Modeling

Constant curvature, Euler-Bernoulli beam



Constant curvature assumption

Assuming uniform actuator torques and

Manipulator joint angle i

$$\theta_i = 2\varphi/(n+1), \quad \forall i = 1 \dots n.$$

Bend angle

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Webster III, Robert J., and Bryan A. Jones. "Design and kinematic modeling of constant curvature continuum robots: A review." *The International Journal of Robotics Research* 29.13 (2010): 1661-1683. **Modified from figure 3.**

Euler-Bernoulli beam assumption under gravity

Successive joint angles decrease exponentially.





Data Collection & Model Calibration





Torsional spring coefficient

- Limb was rotated 90°, left to bend under gravity
- Simplified dynamics:

 $k\boldsymbol{\theta}^{eq} + \mathbf{f_g}(\boldsymbol{\theta}^{eq}) = \mathbf{0}$

- Bend angle measured at static equilibrium
- Constant curvature does not apply; use EB kinematics

$$\theta_i^{eq} = \lambda \varphi^{eq} \frac{(N-i+1)^2}{\sum_{j=1}^N (N-j)^2} = \lambda \varphi^{eq} b_i$$



$$U_g(\boldsymbol{\theta}^{eq}) = mg \sum_{i=1}^n \mathbf{r}_i(\boldsymbol{\theta}^{eq}) \cdot \mathbf{E}_1$$
$$\mathbf{f}_g(\boldsymbol{\theta}^{eq}) = -\nabla_{\boldsymbol{\theta}} U_g(\boldsymbol{\theta}^{eq})$$
$$k = -\boldsymbol{\theta}^{eq} \setminus \mathbf{f}_g(\boldsymbol{\theta}^{eq})$$

Damping coefficient

- Release limb tip from bent position and collect data from damped oscillation
- Dynamics:

 $\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + k\boldsymbol{\theta} + \sigma\dot{\boldsymbol{\theta}} = \mathbf{0}.$

- CC does not apply.
- EB difficult to apply without forces.
- Instead, nested optimization:
 - Fit a damped sinusoid to bend angle trajectory
 - Find the damping coefficient which produces the closest fit

 Algorithm 1: Nested optimization to find σ by matching the damping observed in hardware data $\varphi_{1...t}^d$.

 1 Procedure fitDS $(\varphi_{1...t}) \rightarrow \sigma^*$:

 2
 $\zeta^*, \omega_n^* \leftarrow \arg \min ||\varphi_{1...t} - Ae^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2 t} + \phi) + b||_2^2$

 3
 $\sigma^* \leftarrow \zeta^* \omega_n^*$

 4 Procedure fitData $(\varphi_{1...t}^d) \rightarrow \sigma_{mdl}^*$:

 5
 $\sigma_d^* \leftarrow \text{fitDS}(\varphi_{1...t}^d) \rightarrow \sigma_{mdl}^*$

 6
 $\sigma_{mdl}^* \leftarrow \arg \min ||\sigma_d^* - \text{fitDS}(\varphi_{1...t}^{mdl}|\sigma_{mdl})||_2^2$



Thermal actuator calibration setup

• Collect data under three conditions:

$$\mathbf{f}(\mathbf{T}^{eq}) \neq \mathbf{0}, \dot{\varphi} = 0, \mathbf{\dot{V}} = \mathbf{0}$$

- 1. Actuated,
- 2. Static equilibrium,
- 3. Thermal equilibrium.

• Then make three assumptions:

1.
$$\mathbf{V}^{eq} = \mathbf{T}^{eq}$$

2.
$$k\boldsymbol{\theta}^{eq} = \frac{2k\varphi^{eq}}{(n+1)} \mathbb{1}_{\mathbf{n}} = \left[\beta_r (T_r^{eq} - T_0) - \beta_l (T_l^{eq} - T_0)\right] \mathbb{1}_{\mathbf{n}}$$

3.
$$\mathbf{V}^{eq} \neq 0 \Rightarrow \varphi^{eq} \neq 0.$$

Thermal actuator heat transfer coefficients

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• From assumptions 1 & 2,

$$T_j^{eq} = b\varphi^{eq} + T_0, \quad b = \frac{V^{eq} - T_0}{\varphi^{eq}}$$

• Use relationship to roughly scale the rest of the time series:

$$T_{j,1\ldots t} \hspace{0.1 in} = \hspace{0.1 in} b \varphi_{1\ldots t} \hspace{0.1 in} + \hspace{0.1 in} T_{0}$$

• With T, V, and D known, fit system of ODEs using Julia DiffEqParamEstim.jl package

$$\dot{T} = a_1(T - T_0) + a_2 D,$$

 $\dot{V} = a_3(V - T),$

Thermal actuator force coefficients

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3.
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• From assumption 2, observe

$$\beta_j = [k/(T_j^{eq} - T_0)]\theta_i^{eq}.$$

• Take all such points, stack in vector:

$$\Gamma_{j}^{eq} = \begin{bmatrix} \left(k/(T_{j1}^{eq} - T_{0}) \right) \theta_{i1}^{eq} \\ \vdots \\ \left(k/(T_{jt}^{eq} - T_{0}) \right) \theta_{it}^{eq} \end{bmatrix}$$

• Then compute directly:

$$eta_j = \mathbb{1}_{\mathbf{t}} ig \Gamma_j^{eq}$$

Good model calibration agreement



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Trajectory optimization





Optimization setup

Quadratic-cost optimization

- Q (tracking error penalties) only cares about positions
- R (control penalties) included to minimize control signals

Constraints:

- 1. Initial condition
- 2. System dynamics
- 3. Valid controls
- 4. Safe temperatures
- 5. Warm operation

Optimized feasible Optimized open-loop
state trajectory Control trajectory Tracking error

$$\mathbf{x}_{1...N}^{*}, \mathbf{u}_{1...N}^{*} = \arg\min_{\mathbf{x},\mathbf{u}} \frac{1}{2} \sum_{k=1}^{N-1} \left(\mathbf{\tilde{x}_{k}}^{\top} \mathbf{Q} \mathbf{\tilde{x}_{k}} + \mathbf{u_{k}}^{\top} \mathbf{R} \mathbf{u_{k}} \right)$$

Heres
 $+ \frac{1}{2} \mathbf{\tilde{x}_{N}}^{\top} \mathbf{Q}_{N} \mathbf{\tilde{x}_{N}}$
s.t. $\mathbf{x_{1}} = \mathbf{x}_{init}$
 $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_{k}, \mathbf{u}_{k})$
 $\mathbf{0} \le \mathbf{u}_{k} \le \mathbb{1}_{2}$
 $\mathbf{T}_{k} < T_{max} \mathbb{1}_{2}$
 $\mathbf{T}_{k} > T_{warm} \mathbb{1}_{2}$ $\forall k > k_{warm}$.

Optimization results





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Open-loop Tracking Results



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• Hardware tracking good, but improvement is needed



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- Hardware tracking good, but improvement is needed
- "Soft" startups difficult



- Hardware tracking good, but improvement is needed
- "Soft" startups difficult
- Some tracking bias observed



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- Some tracking bias observed
- Temperature does not track well
 - Timescale of original validation too long



 Thermal model fit motion dynamics well ("T") but not observations ("V")



Teach -and- Repeat





Teach-and-repeat results



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- Improvements to SMA force model
 - At least rudimentary modeling of hysteresis

Shape memory alloy hysteresis [1]



[1] Liang, Chen, and Craig A. Rogers. "One-dimensional thermomechanical constitutive relations for shape memory materials." *Journal of intelligent material systems and structures* 8.4 (1997): 285-302.



- Improvements to SMA force model
 - At least rudimentary modeling of hysteresis
 - Better stress mapping, especially outside linear region

Shape memory alloy hysteresis [1]



SMA unified constitutive relation [2] $\tau - \tau_0 = G(\xi)(\gamma - \gamma_0) + \frac{\Theta}{\sqrt{3}}(T - T_0) + \frac{\Omega(\xi)}{\sqrt{3}}(\xi - \xi_0)$

[1] Liang, Chen, and Craig A. Rogers. "One-dimensional thermomechanical constitutive relations for shape memory materials." *Journal of intelligent material systems and structures* 8.4 (1997): 285-302.

[2] Cheng, Shing Shin, Yeongjin Kim, and Jaydev P. Desai. "Modeling and characterization of shape memory alloy springs with water cooling strategy in a neurosurgical robot." *Journal of intelligent material systems and structures* 28.16 (2017): 2167-2183.

- Improvements to SMA force model
 - At least rudimentary modeling of hysteresis
 - Better stress mapping, especially outside linear region
- Improvements to SMA temperature model
 - Higher priority on matching measurement temperature



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- Investigate use of ML models to model residual error



- Improvements to SMA force model
 - At least rudimentary modeling of hysteresis
 - Better stress mapping, especially outside linear region
- Improvements to SMA temperature model
 - Higher priority on matching measurement temperature
- Investigate use of ML models to model residual error and bias
- Extend to multi-link system
 - Full Horton robot is composed of five links (three legs, two shoulders)





Main contributions			



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Development and calibration of simplified models for thermal-actuation and soft manipulator dynamics.				
\mathbf{E}_{2}				



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Development and calibration of simplified models for thermal-actuation and soft manipulator dynamics.	Design of feasible trajectories and open-loop controls through optimization.			
\mathbf{E}_{2} \mathbf{E}_{1} \mathbf{e}_{3} \mathbf{e}_{3} \mathbf{e}_{3}	Optimization: Input vs. Feasible Result, Trajectory A $ \begin{array}{c} $			

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Main contributions				
Development and calibration of simplified models for thermal-actuation and soft manipulator dynamics.	Design of feasible trajectories and open-loop controls through optimization.	Validation in hardware using an SMA-actuated soft manipulator.		
\mathbf{E}_{2}	Optimization: Input vs. Feasible Result, Trajectory A $ \begin{array}{c} $			

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